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Multiobjective Nonlinear Model Predictive Control of Forestry Problems

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Abstract: The use of rigorous computational tools is necessary to control forest pollution and minimize carbon dioxide emissions. In this work, a rigorous multi objective nonlinear model predictive control strategy is adopted on three different forestry models. The optimization was performed with the optimization language PYOMO in conjunction with the state-of-the-art optimization solvers IPOPT. The globality of the solutions was confirmed with the global optimization solver BARON. The optimum profiles generated show that this strategy is effective in minimizing the carbon dioxide emissions, and forest pollutants maximizing the forest biomass density. The control of non-wood-based industrial activity is beneficial to keep the depletion of forestry as low as possible and minimize the emission of unwanted carbon dioxide into the atmosphere.

Keywords: Multiobjective, Optimization, Industries, Forest

The presence of forests and forested areas is very beneficial for the health and well-being of humanity. The rise of industrial and residential areas arising from population growth has led to deforestation and the destruction of a lot of greenery. This coupled with industrialization has led to the production of a lot of carbon dioxide which is detrimental to human health. A lot of computational work has been performed studying the dynamics of various situations involving forestry, specifically optimal control. However, all the work so far involves single-objective optimal control. Computational techniques have been used by several researchers to model forest depletion caused by population increase and industrial development. Shukla and co-workers (1989, 1996, 2003, 2006, 2009) performed computational work on effects of population growth, industrialization and pollutant formation on forest density. Freedmanand Shukla (1991) developed models for the effect of toxicant in single-species and predator-prey systems.Shukla and co-workers (2003) studied the effects of primary and secondarytoxicants on renewableresources. Similarly, Naresh et al (2006) computationally investigated the effect of an Intermediate toxic product formed by uptake of a toxicant on plant biomass. Shukla et al (2009) modelled the survival of a resource dependent population, studying the effects of toxicants (pollutants) emitted fromexternal sources as well as formed by its precursors. Dubey and Narayanan (2010) worked out the interactions between industrialization, population and pollution. Shah et al (2017) performed optimal control studies for the spread of pollutants through forest resources. Irma Fitria et al (2021) modeled the dynamics of CO_2 emission, forest area, and industrialization and used optimal control to minimize the carbon dioxide emission. Betancourt et al (2024) observed the changes in timber yield of commercial tree species in the eastern Brazilian Amazon based on 33 years of inventory data. This paper deals with the performance of multiobjective nonlinear model predictive control (MNLMPC) tasks on the three models described by Dubey et al (2009) and Shah et al (20170 and Irma Fitria et al (2021). First, the three problems involving forestry are described. This is followed by a discussion of the MNLMPC strategy. Then the MNLMPC results for each of the three problems are presented along with a detailed discussion. This is followed by a summary of the results and the conclusions.

MATERIAL AND METHODS

Problem 1: In this problem (Irma Frita el al 2021) the variables are:

- X the carbon dioxide concentration,
- I the industrial existence,
- Z, the forest presence.

The parameter values are

- r, the growth rate of carbon dioxide concentration, =0.15((1/year),
- s, the carrying capacity of carbon dioxide=700(ppm),
- α, the emission reduction due to forest resources, =0.06 (ppm ha^{-1} $yeat^{-1}$)

- h_1, the emission growth rate due to industrialization=0.8 (ppm/year),
- h_2, the natural depletion rate of forest resources=1(1/year),
- β(industrialization growth rate) =0.1(ha^{-1}),
- h_3 (depletion rate of forest resources) =0.02(1/year),
- γ, the natural forest growth rate =0.07 (1/year).

The control variables are u_1 (the optimal control of the reforestation) and u_2 (the optimal control of government policy). The equations are (1-3)

$$\frac{dX}{dt} = rX(1-\frac{X}{s}) - \alpha Z + h_1 I \quad (1)$$

$$\frac{dZ}{dt} = (\gamma + u_1 - h_1 - h_3 I)Z \quad (2)$$

$$\frac{dI}{dt} = (\beta h_3 Z - u_2)I \quad (3)$$

Problem 2: In this problem (Dubey et al 2009) the variables are

- B (t) is the density of resources biomass density,)1/ha)
- N(t) is the cumulative density of populations, (1/ha)
- P(t) is the population density pressure, (1/ha)
- I(t) the industrialization density. (1/ha)

The parameters are

- s is the intrinsic growth rate = 34 (1/year)
- L is the carrying capacity = 40 (1/ha)
- S_0 is the resource biomass natural depletion rate coefficient=1
- r_0 is the population natural depletion rate coefficient=10
- β_1 the population cumulative density growth rate because of resources 0.01 (1/ha)
- β_2 the depletion rate coefficient of the resource biomass density due to population=7 (1/year)
- λ, The population pressure growth rate coefficient =5 (1/year)
- λ_0, the natural depletion rate coefficient =4 (1/year)
- θ the depletion rate coefficient caused in augmenting industrialization =8,(1/year)
- S_1 is the depletionrate coefficient of the biomass density resulting from industrialization =4, (1/year)
- π_1, industrialization growth rate because of resource= 0.005 (1/ha)
- π is the industrialization growth rate because of population pressure= 0.001. (1/ha)

θ_0 is the control coefficient because of governmental regulations and is the control variable. The equations involved are (4-7)

$$\frac{dB}{dt} = s(1-\frac{B}{L})B - s_0 B - \beta_2 NB - s_1 IB \quad (4)$$

$$\frac{dN}{dt} = r(1-\frac{N}{K})N - r_0 N + \beta_1 NB \quad (5)$$

$$\frac{dP}{dt} = \lambda N - \lambda_0 P - \theta P \quad (6)$$

$$\frac{dI}{dt} = \pi\theta P + \pi_1 s_1 IB - \theta_0 I \quad (7)$$

Problem 3: In this (Shah et al , 2017) variable are

- W is the density of wood-based industries (1.ha)
- F is the density of forest resources(1.ha)
- I is the density of non wood based industries (1.ha)
- PI represents the pollutants through non-wood-based industries (ppm)
- PW is the pollutants through -wood-based industries (ppm) (8-12)

$$\frac{dF}{dt} = B - (\beta W + g)F - \beta_1 FW + \varepsilon_1 P_W - \gamma_1 F - \gamma_2 F + \varepsilon_2 P_I + u_1 W - \mu F \quad (8)$$

$$\frac{dW}{dt} = (\beta W + g)F + \beta_1 FW - \delta_1 W + \delta_2 I - \eta_1 W - u_1 W - u_2 W - \mu W \quad (9)$$

$$\frac{dI}{dt} = QI + \delta_1 W - \delta_2 I - \eta_2 I - u_3 I - \mu I \quad (10)$$

$$\frac{dP_W}{dt} = \eta_1 W - \varepsilon_1 P_W + \gamma_1 F + u_2 W - \mu_W P_W \quad (11)$$

$$\frac{dP_I}{dt} = \eta_2 I - \varepsilon_2 P_I + \gamma_2 F + u_3 I - \mu_I P_I \quad (12)$$

The parameter values are

- B the rate of compactness degree of forest resources=100 (1/year)
- Q The constant rate of resources provided to non-wood based industries (1/year)
- which does not depend on forest resources = 0.6 (1/year)
- g Migration of wood based industries to the forest region which directly depends on the density of forest resources =0.8 (1/year)
- β The depletion rate of forest resources due to wood based industries =0.04 (1/year)
- β_1 The growth rate of wood based industries due to forest resources =0.003 (1/year)
- μ The natural depletion rate =1 (1/year)
- μ_w The natural depletion rate of pollutants emitted from wood based industries=1 (1/year)
- μ_1 The natural depletion rate of pollutants emitted from non-wood based industries =1 (1/year)
- δ_1 he rate of competition effects of I on W =0.5 (1/year)
- δ_2 The rate of competition effects of W on I=0.3 (1/year)
- ε_1 The loss of pollutants generated by wood-based industries due to forest resources =0.02 ppm

- ε_2 The loss of pollutants generated by nonwood-based industries due to forest resources =0.01 ppm
- γ_1 The depletion rate of forest resources caused by the pollutants generated through wood-based industries =0.5 (1/year)
- γ_2 The depletion rate of forest resources caused by the pollutants generated through nonwood-based industries =0.5 (1/year)
- η_1 The growth rate of pollutants generated by wood-based industries =0.1 (1/year)
- η_2 The growth rate of pollutants generated by wood-based industries =0.7 (1/year)

The control variables in the problem are

- u1 is the rate that decreases wood-based industries to control the usage of forest resources.
- u2 is the control rate that decreases pollutants due to wood-based industries
- u3 is the control rate that decreases pollutants due to non-wood-based industries

MNLMPC (Multiobjective Nonlinear Model prediotive control) method: The multiobjective nonlinear model predictive control strategy (MNLMPC) method was first proposed by Flores Tlacuahuaz (2012) and used by Sridhar [2019]. This method does not involve the use of weighting functions, nor does it impose additional constraints on the problem unlike the weighted function or the epsilon correction method (Miettinen, 1999). For a problem that is posed as

$$\begin{aligned} &\min J(x,u) = (x_1, x_2 \ldots. x_k) \\ &subject\ to\ \frac{dx}{dt} = F(x,u) \\ &h(x,u) \le 0 \\ &x^L \le x \le x^U \\ &u^L \le u \le u^U \end{aligned} \tag{13}$$

The MNLMPC method first solves dynamic optimization problems independently minimizing/maximizing each x_i individually. The minimization/maximization of x_i will lead to the values x_i^*. Then the optimization problem that will be solved is

$$\begin{aligned} &\min \sqrt{\{x_i - x_i^*\}^2} \\ &subject\ to\ \frac{dx}{dt} = F(x,u) \\ &h(x,u) \le 0 \\ &x^L \le x \le x^U \\ &u^L \le u \le u^U \end{aligned} \tag{14}$$

This will provide the control values for various times. The first obtained control value is implemented and the remaining discarded. This procedure is repeated until the implemented and the first obtained control value are the same. In optimization package in Python, Pyomo differential equations are automatically converted to a Nonlinear Program (NLP) using the orthogonal collocation method (Biegler 2007) . The Lagrange-Radau quadrature with three collocation points is used and 10 finite elements are chosen to solve the optimal control problems. The resulting nonlinear optimization problem was solved using the solvers IPOPT(Wachter et al 2006) and confirmed with Baron (Tawarmalani 2005) To summarize the steps of the algorithm were:

1. Minimize/maximize x_i subject to the differential and algebraic equations that govern the process using Pyomo with IPOPT and Baron. This will lead to the value x_i^* at various time intervals t_i. The subscript *i* is the index for each time step.
2. Minimize subject to the differential and algebraic equations that govern the process using Pyomo with IPOPT and Baron. This will provide the control values for various times.
3. Implement the first obtained control values and discard the remaining.

Repeat steps 1 to 4 until there is an insignificant difference between the implemented and the first obtained value of the control variables.

RESULTS AND DISCUSSION

Problem 1:_The multiobjective nonlinear model predictive control problem involves the maximization of the forest presence, and at the same time minimizing the industrial experience and carbon-dioxide emissions. This is equivalent to maximizing ΣZ_i while minimizing ΣX_i and ΣI_i. ΣZ_i The maximization of results in a value of 100, while the minimization of ΣX_i and ΣI_i results in values of 0 for each. The overall minimization objective function will be $(\Sigma I_i\text{-}100)^2$ + $(\Sigma X_i\text{-}0)^2$ +$(\Sigma I_i\text{-}0)^2$ subject to the equations governing this problem. The MNLMPC strategy will ultimately reduce the emanation of carbon dioxide X, with the increase in forestation and the reduction of industrial density(Figs 1a-1e).

Problem 2:_The density of resources biomass density, is maximized and the population density pressure is minimized. This is equivalent to maximizing ΣB_i and minimizing ΣP_i / The maximization of ΣB_i results in a value of 40.016, while the minimization of ΣP_i results in a value of 0. The multiobjective optimization results in the minimization of $(\Sigma B_i - 40.016)^2$ + $(\Sigma P_i - 0)^2$ subject to the equations governing this problem. This strategy results in the increase in biomass density and the reduction in industrial density (Fig. 2a-2c).

Problem 3:_Here, P_w, P_I are both minimized while F, W and I

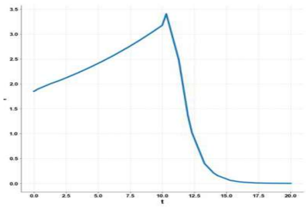

Fig. 1a. I vs t diagram for problem 1

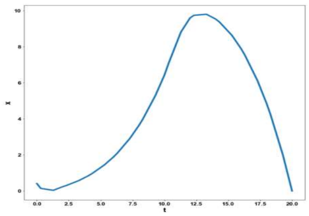

Fig. 1b. I vs t diagram for problem 1

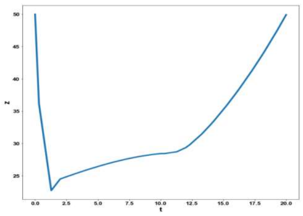

Fig. 1c. z vs t diagram for problem 1

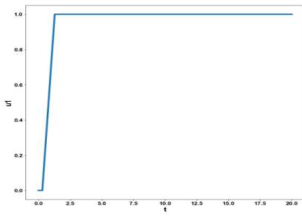

Fig. 1d. u1 vs t diagram for problem 1

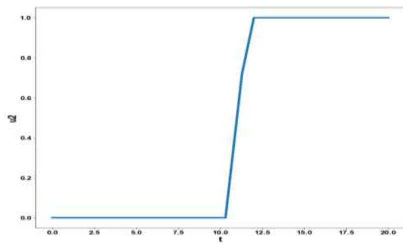

Fig. 1e. u2 vs t diagram for problem 1

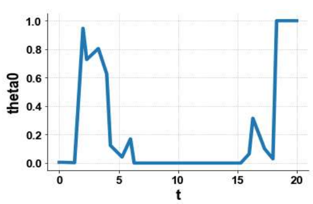

Fig. 2a. theta0 vs t for problem 2

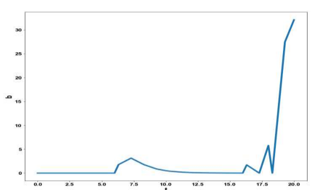

Fig. 2b. b vs t for problem 2

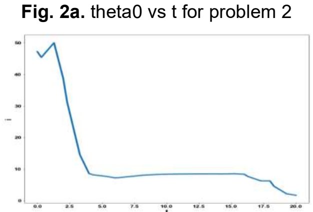

Fig. 2c. i vs t for problem 2

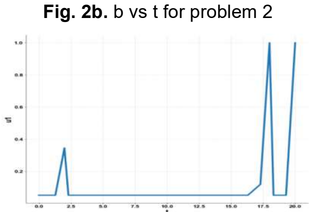

Fig. 3a. u1 vs t for problem 3

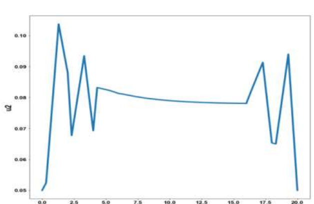

Fig. 3b. u2 vs t for problem 3

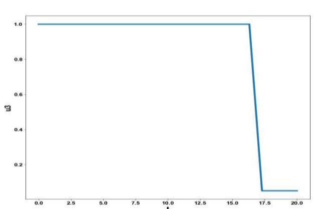

Fig. 3c. u3 vs t for problem 3

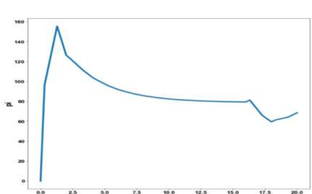

Fig. 3d. (pi vs t problem 3)

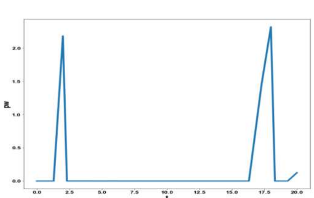

Fig. 3e. (pw vs t problem 3)

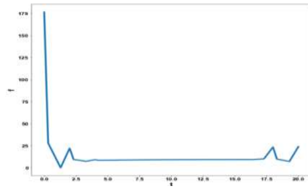

Fig. 3f. (f vs t problem 3)

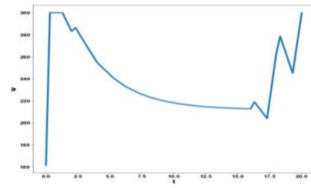

Fig. 3g. (w vs t problem 3)

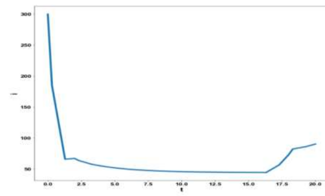

Fig. 3h. (i vs t problem 3)

are maximized. The minimization of H P_w, P_I lead to values of 0 and 58.73, while the maximization of F, W and I lead to values of 314.19, 600 and 411.71. The objective function for the multiobjective optimization will be

$(P_w - 0)^2 + (P_I - 58.73)^2 + (F - 314.19)^2 + (W - 600)^2 + (I - 411.71)^2$ subject to the equations representing this problem. The various profiles are shown in figures 3a-3h

Indicate the reduction of non-wood based industries causes a reduction in forest pollution.

In each of the three problems the variable I ultimately reduces with time using this strategy. The multiobjective nonlinear model predictive control strategy demonstrates that the minimization of the non-wood based industrial activity reduces carbon dioxide emissions, and other forest pollutions and increases biomass density in a given area.

CONCLUSIONS

A rigorous multiobjective optimal control procedure that does not involve additional constraints or weighting functions is used on forestry models. The main finding is that to increase biomass density, carbon dioxide emission and forest pollutants must be minimized . It is seen that controlling the non-wood industrial activity is essential to achieve these pbjectives.

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